A Formal Model for Sustainable Vehicle-to-Grid Management

Mohammad Ashiqur Rahman, Fadi Mohsen, and Ehab Al-Shaer
Department of Software and Information Systems
University of North Carolina at Charlotte, USA
{mrahman4, fmohsen, ealshaer}@uncc.edu

ABSTRACT

The Vehicle-To-Grid (V2G) technology allows plug-in electric vehicles (PEVs) to act like an energy provider besides being a consumer. A PEV, being connected to the smart grid, can either charge its battery by consuming electricity from the grid or discharge the stored electricity from the battery to the grid. It can also participate in the frequency regulation service of the grid. Executing the aforementioned operations in a non-controlled fashion may come with problems on the grid functionality. For safe and sustainable functioning of the grid, controlling the operations is very crucial. In this paper, we are offering an approach for vehicle-to-grid management using constraint-based formal modeling. The approach is centered around an aggregator that collects all the involved parties' constraints and preferences. The aggregator then finds a management plan, i.e., a schedule of V2G services for the PEVs by satisfying the given constraints besides its own requirements. We apply *satisfiability modulo theories* (SMT) to synthesize the schedule as a satisfaction of the constraints. Our evaluation results show that the formalization can be efficiently solved for problems with thousands of PEVs.

Categories and Subject Descriptors

J.m [**Computer Applications**]: Miscellaneous;
F.4.m [**Theory of Computation**]: Mathematical Logic and Formal LanguagesMiscellaneous

General Terms

Management

Keywords

Smart Grid; Plug-in Electric Vehicle; Vehicle-to-Grid; Formal Model

1. INTRODUCTION

Energy distributors are facing the problem of achieving the right balance between production rate and consumption rate. Energy consumption varies greatly over the years, months, weeks and even the days. For instance, the peak usage of electricity (between 6PM and 7PM in the U.S.A.) is higher than the off-peak periods. The peak value is very crucial to energy providers as they need to generate power in a rate that is higher than the peak value. Researchers have addressed the issue of minimizing the cost of generation based on active communications between providers and consumers [1]. With the trend of taking the power grid in North America into next level (smart grid) by integrating the sensing, communications, and control technologies, Plug-in Hybrid or full Electric Vehicles (PHEV or PEV) appear on the surface [2].

Due to the improvement of the technology, PEVs now have bigger batteries and a plug-in cord to access grid power. The PEVs, while plugged-in, can be used as a small and distributed storage mechanism by the grid. As the number of electric cars increases, the combined storage could provide different electrical (e.g., energy generating capacity and energy supply) and ancillary services (e.g., frequency regulation and voltage control) for the grid. Therefore, a PEV can play two different roles: either a consumer or a provider. When a PEV charges its battery, it acts as a consumer. As a provider, a PEV can supply its stored charge to the grid or can participate in frequency regulation for the grid. Since a PEV has very small capacity compared to the need of the smart grid, a large number of PEVs are combined to offer useful services to the grid. The concept of *aggregator* is introduced for the Vehicle-To-Grid (V2G) management. PEVs subscribe to an aggregator according to their availability, while the aggregator does the management of the V2G services, i.e., energy consumption, supplying (i.e., selling) energy to the grid, and participating in the ancillary services, such as frequency regulation. An aggregator creates the bridge between the energy provider's requirement and each subscribed PEV's interest.

A PEV, collectively with other PEVs, can play a vital role in the energy sustainability as it can play the role of a provider, beside being a consumer. As a consumer a PEV pays to the grid, while as a provider it earns money. The ultimate cost or profit depends on the time-varying pricing model. Moreover, unbounded charging, discharging, or participation in frequency regulation is not safe for the grid, especially in terms of grid's capacity or need. Therefore, a plan of operations is essential for the optimal performance. However, the process of achieving a V2G management plan, by solving

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the critical constraints for the safe and sustainable functionality of the grid, is very challenging. The complexity of the problem arises from the number of stakeholders (i.e., a large number of PEVs, multiple aggregators, the energy provider), the number of constraints for each stakeholder, their conflicting interests or goals, and time-varying price models. Due to the large number of the participating vehicles, it is crucial to have an automated model that can efficiently provide a V2G management plan for an aggregator.

In this paper, we model the *V2G management* as a constraint-satisfaction problem and use Satisfiability Modulo Theories (SMT) [3] to obtain satisfiable solutions. We consider an aggregator-based V2G model and our aim is to synthesize a management plan for the aggregator for executing the V2G operations on the PEVs taking the constraints of the participants into consideration. In this synthesis process, the aggregator must know the number of subscribers, their charging rates, their subscription periods, i.e., when they are going to connect to the grid and leaving the grid, etc., besides the constraints given by the grid. The aggregator needs to know the information and requirements by a time ahead of the period for which it is preparing a management plan. We assume that a PEV provides the aggregator with the necessary information for the designated upcoming period.

This paper is organized as follows: We describe the background of the V2G system and the motivation for V2G management in Section 2. We present the formal modeling of the V2G management problem along with an illustrative example in Section 3. We present evaluation results in Section 4. We discuss the related works in Section 5. We write the summary and the future works in Section 6.

2. BACKGROUND AND MOTIVATION

We first describe the state of the art characteristics of the V2G system in this section. Next, we define the V2G management problem. Finally, we discuss the motivation of using SMT for solving our model.

2.1 V2G: State of the Art

Plug-in electric vehicles (PEVs) have become a reality and expected to grow largely in the near future. The number of PEVs in the United States is expected to exceed 10 million by the year 2020 [4]. PEVs have to be charged regularly and according to their usage pattern. In fact, uncontrolled charging can easily increase the peak value and, moreover, can increase the energy consumption uncertainty. However, the V2G system allows the flow of energy from the PEVs to the grid [5,6]. The energy provider can utilize the combined storage of a number of PEVs in regulating its production rate, which is known as frequency regulation. In other cases, the stored electricity can be sold to the energy provider, which can be used for lightening up houses, streets and universities instead of producing that amount, especially during the peak demand periods, thus reducing the cost. The PEVs' owners can take advantage of this V2G capability by charging the batteries during off-peak periods, while selling stored energy to the grid when the demand is high.

V2G Operations. The batteries of PEVs can consume power from the grid, provide (sell) power to the grid, or participate in frequency regulation. We call these three operations as *charging*, *discharging* (or *selling stored energy*), and *participating in frequency regulation*, respectively. However, frequency regulation is actually a process of frequent charging and discharging according to the state of the power supply and load, which is required to keep the frequency at a stable point. However, charging and discharging operations cannot be overlapped with participating in frequency regulation. For example, when a vehicle battery is being charged for its own sake, it plays a role as an added load on the grid, which is also affecting the required regulation amount. If the load suddenly disappears (or be adjusted) while the regulation control is trying to accommodate the load, it would create a conflict in balancing between supply and demand. The detail explanation will be found in [7].

The payment of selling stored electricity is usually equal to the current price of the electricity. The regulation service is paid mainly by its storage capacity rather than the actual state of the charge (SOC) of the storage. Because, the fluctuations of power changes between positive and negative are almost evenly distributed [8], i.e., the energy delivered and the energy absorbed are almost equal over a long-term regulation. Therefore, in this literature we assume that participating in frequency regulation does not change the SOC of the battery.

Why do we need the aggregator? There is a gap between the provider's (PEV's) power capacity, which is 10-20 kW, and the consumer's (energy provider's) requirement, which is measured in MW basis. Because of that an aggregator is necessary to help organize the activity of responding to the need of the energy provider as well as a larger number of PEVs. Therefore, the standard is as follows: PEVs (i.e., the owners of the car) subscribe to an aggregator that organizes the PEVs' operations satisfying the goals. Fig. 1 shows an example of such an aggregator based architecture. Multiple aggregators may simultaneously serve an energy provider with V2G services. We assume that the energy provider specifies its requirements distinctly to each aggregator. Therefore, an aggregator's management plan depends only on its subscribers and their requirements along with the provider's requirements.

2.2 V2G Management Problem

The goal of the V2G management problem is to find an efficient choice of actions for the vehicles at different times of their subscription. The choices of actions includes (i) *charging*, (ii) *discharging*, and (iii) *participating in frequency regulation*. The vehicles subscribe to the aggregator and the aggregator manages the vehicles' actions. The actions depends on different constraints introduced by the participating parties or users, i.e., the energy provider and the subscribed vehicles. The aggregator has its own constraints as well, tailored towards its interest.The V2G management problem is defined as the scheduling of actions that satisfies the users' requirements, meanwhile meeting aggregator's targeted goals (benefits).Given that energy price varies from a time slot to another, users' submitted constraints, and grid requirements. V2G management is a complex and challenging problem. Finding a compromising solution to this problem is crucial, since an aggregator must fulfill all those constraints altogether to guarantee customer satisfaction, besides satisfying its own profit. A successful aggrega-

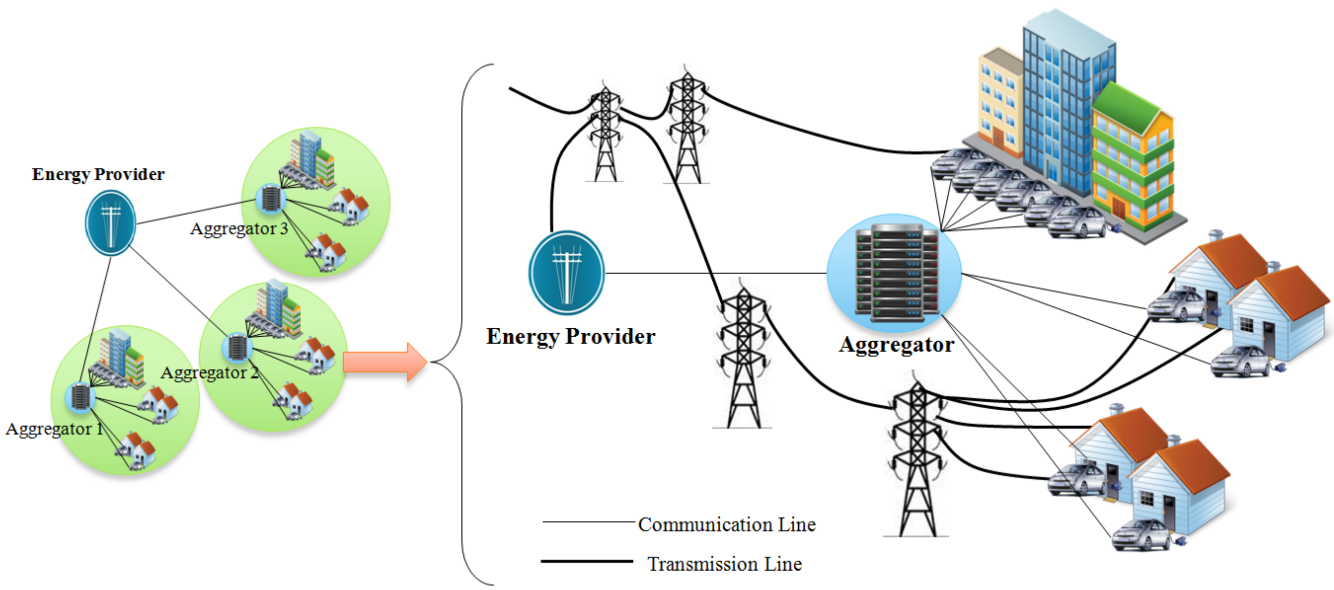


Figure 1: A simplified architecture showing a V2G system through an aggregator.

tor needs to keep the balance between its goals and other parties' goals and interests. The interests could mean actual profit or providing good service (reputation).

If the aggregator wants to find the management plan that will provide the optimal payoff, e.g., the maximum benefit to each of the aggregator and PEVs, the solution to the V2G Management Problem becomes NP-complete. In the following, we provide Theorem 1 to prove this.

THEOREM 1. *The optimal vehicle-to-grid management problem is NP-complete.*

PROOF. We can reduce the equal sum partition problem to the Vehicle-to-Grid management problem. Suppose we have an instance of equal sum partition problem with a set of natural numbers $S = \{a_1, a_2, \ldots, a_n\}$ and $\sum_{i=1}^{n} = K$. The equal sum partition problem asks one to find a subset S' of S such that the sum of numbers in S' is $K/2$. Now we can construct an instance of the Vehicle-to-Grid management problem, where we have two time slots and n vehicles $v_1, v_2, \ldots, v_n$. The battery capacity (i.e., electricity storage capacity) of vehicle v_i $(1 \leq i \leq n)$ is a_i. The upper bound of the amount of discharged electricity for every time slot is $K/2$, and the minimum limit for the energy stored in the battery of every vehicle is 0. The payoff for discharging is exactly the amount of discharged electricity. The cost for frequency regulation is 0. Now we can see that, to get the maximum payoff, one needs to arrange a subset of vehicles for discharging in every slot, and the total amount of discharging in every slot should be $K/2$. This is equivalent to find a solution for the original equal sum partition problem. It is known that the equal sum partition problem is NP-complete [9], so the Vehicle-to-Grid management problem is also NP-complete. □

Therefore, in this work we model the V2G management as a satisfiability problem, particularly an SMT problem.

2.3 Efficiency in Using SMT

SMT is a powerful tool to solve constraint satisfaction problems that arise in many diverse areas including software and hardware verification, test-case generation, scheduling, planning, graph problems, etc. [3]. SMT is the problem of determining whether a formula is satisfiable or not. For example, the SMT instance with the following two constraints is satisfiable with the assignments of $x = 0$ and $y = 0$:

$$(x + y < 2) \vee (x - 2y > 0)$$

$$x \leq 1$$

SMT provides a much richer modeling language than is possible with SAT [10]. In SMT, complex Boolean logics are replaced by first order logics using a variety of underlying theories, e.g., the theory of equality, linear arithmetic, difference logic, etc. Modern SMT solvers can check formulas with thousands of variables, and millions of clauses [11]. If an SMT instance cannot be satisfied, one needs to relax the constraints (e.g., in our case, decreasing the expected benefit of the aggregator) to find a satisfiable solution.

3. FORMAL MODEL OF V2G MANAGEMENT

In this section, we formalize the V2G Service Management Problem as a satisfaction of a number of constraints. We start the section by describing the V2G system.

3.1 V2G System

We define a V2G system consisting of a large number of plug-in electric vehicles, an aggregator and a power grid utility (i.e., energy provider). Figure 1 shows a simplified

Table 1: The Notations and their Meanings

Notation	Definition
$\mathbb{V}$	The set of Plug-in Electric Vehicles.
v	A PEV, such that $v \in \mathbb{V}$.
$\mathbb{T}$	The set of time slots.
t	A time slot, such that $t \in \mathbb{T}$.
ST_v	The time when v starts its subscription to the grid.
ET_v	The time when v ends its subscription to the grid.
B_v	The capacity of v's battery.
IC_v	The initial state-of-charge (SOC) when v starts its subscription.
EC_v	The required SOC when v ends its subscription.
R_v	The battery charging/discharging rate.
F_v	A Boolean parameter to denote that whether the vehicle would like to participate in frequency regulation.
$\mathbb{O}$	The set of operations that a battery can perform.
$O_{v,t}$	The operation chosen for v's battery at time slot t, such that $O_{v,t} \in \mathbb{O}$.
PE_t	The price of an unit of electricity during t.
PF_t	The price for an unit capacity of battery that is used for frequency regulation during t.
XC_t	The maximum electricity that can be used in total for charging the vehicles during t.
ND_t	The minimum electricity (in total) less than which can not be discharged to the grid by the vehicles during t.
XD_t	The maximum electricity that can be discharged to the grid in total by the vehicles during t.
NF_t	The minimum total battery capacity (of the vehicles) required for frequency regulation during t.
XF_t	The maximum total capacity more than which will not paid though that is also assigned for frequency regulation during t.
NB	The minimum battery capacity required for a vehicle for subscription.
NR	The minimum charging/discharging rate required for a vehicle for subscription.
$C_{v,t}$	The SOC of v at the end of t.
$VC_{v,t}$	The charge/electricity consumed by v during t.
$VD_{v,t}$	The charge/electricity discharged by v during t.
$VF_{v,t}$	The battery capacity assigned for frequency regulation by v during t.
TC_t	The total charge/electricity consumed by all subscribed vehicles during t.
TD_t	The total charge/electricity discharged by all subscribed vehicles during t.
TF_t	The total battery capacity assigned by all subscribed vehicles for frequency regulation during t.
NP	The minimum payoff of the aggregator.

architecture of a V2G system. The vehicles subscribe to an aggregator to participate in different V2G services. In this model, we consider that a duration of time is divided into number of time slots (e.g., 24 time slots) as shown in Figure 2.

We assume a time-ahead (e.g., day-ahead) information model, i.e., the information about the future for a time period is known. When a vehicle subscribes to the system, it provides the necessary information and requirements to the aggregator. The information includes the following:

- The start and end of subscription period, which is the time during which the vehicle promises to be connected to the grid and follows an aggregator control commands.
- The electric charge capacity of vehicle's battery, i.e., the maximum charge that the battery can store.
- The rate in which the battery can be charged or discharged. We consider the same rate for both charging and discharging.
- The initial stored energy in a battery, i.e., the initial state-of-charge (SOC). The initial means at the very beginning of the subscription period.
- The minimum SOC requirement of the battery at the end of the subscription period, which the vehicle needs for its next travel.
- Whether or not the vehicle would like to participate in the frequency regulation service.

A PEV may not be interested in the frequency regulation support. Each battery has specific life cycles, e.g., 5-10 thousand cycles in the case of full charge (or discharge) and 6-10 times more cycles in the case of small amount of charge (or discharge) [12]. Since frequency regulation needs frequent charging and discharging, though in small amount, it deteriorates the battery life. However, participation in frequency

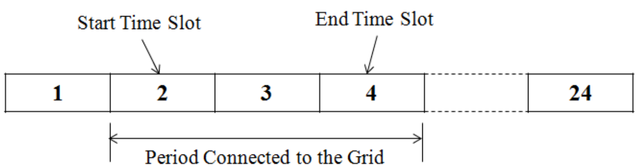


Figure 2: The modeling of time as a collection of slots.

regulation has been proved to be beneficial considering the cost of the battery and the battery wear [12,13]. Still, a vehicle might have different constraints on buying a new one if the battery dies. That is why we consider the participation in frequency regulation as an option chosen by the vehicle upon its subscription. A similar constraint can be taken in the case of selling energy, which is equivalent to a full discharge cycle. Since the discharging events are very infrequent, their impact on the battery is negligible compared to the frequency regulation scenario. Therefore, in this work we do not consider a vehicle's choice on selling energy. It is also worth mentioning that the battery life expectance is not considered directly in the model for several reasons. For instance, PEV batteries vary a lot in quality and performance. It is hard to quantify the effect of charge/discharge on the battery life. Moreover, the aggregator needs to keep track of the battery's charging/discharging history for each PEV.

The energy provider also has a number of requirements that the aggregator should consider during scheduling the vehicles' operations. The following are examples of such requirements:

- The total discharged electricity from PEVs to the grid must fall within a range. Because, a very low amount of energy may not worth the process. On the other side, the grid's transmission system may not be capable to carry a large amount of (extra) energy to the grid.
- The total charge at a particular time slot may have to be less than a threshold value, so that the overall load does not exceed the production/distribution capability.
- The total electricity storage, i.e., accumulated battery capacity of the vehicles participating in frequency regulation, should be more than a threshold capacity value. For maintaining the stability of the grid's demand, the grid requires this minimum level of capacity from the aggregator. The energy provider also may not require very large capacity for frequency regulation. Therefore, there is a maximum aggregated capacity more than that the energy provider may not pay.

Since the electricity demand varies over the day, the above requirements are specified over the time slots. The energy provider provides the aggregator with the electricity prices with respect to the time slots. The electricity price is usually high during those time slots when the electricity demand is high compared to the time slots when the demand is low. The aggregator pays the energy provider for the energy consumption (i.e., charging) with this price. When the aggregator sells electricity, the energy provider pays the aggregator based on the same price. The frequency regulation support is also paid usually according to a time-varying price model.

The aggregator has some requirements of its own. The main requirement is that it wants to receive an optimal payoff, i.e., it wants to maximize the revenue from selling electricity and participating in frequency regulation compared with the price paid for electricity consumption. We consider this as the minimum payoff requirement. One can adjust this requirement till finding the optimal payoff. The way the aggregator pays its subscribers is not covered by our model, assuming that the billing depends on its payoff and vehicles' quality of participation. Moreover, the aggregator can pose some minimum requirements on vehicles upon subscription: the vehicle's battery must have a minimum capacity and a minimum charging/discharging rate. In order to have a correct scheduling, we assume that once a vehicle subscribes to the system, it remains connected to the grid as it has specified. We also assume that in a particular time slot, a vehicle can do only a single function, i.e., either charge, discharge, or participate in frequency regulation.

3.2 System Model

In this subsection, we present the modeling of vehicles, its operations, pricing models, and user requirements.

Plug-in Electric Vehicle. We denote the set of vehicles subscribed for V2G services by $\mathbb{V}$. Each vehicle v ($v \in \mathbb{V}$) has different properties. These are as follows: the starting time (beginning of a time slot) of the subscription, ST_v; the ending time (end of a time slot) of the subscription, ET_v; the battery capacity, B_v (kWh); the stored electric charge, IC_v (kWh); and the battery charging/discharging rate, R_v (kWh per time slot). There is also a parameter, F_v, which denotes whether the vehicle would like to participate in frequency regulation.

Vehicle's Operation. A PEV can do three different operations: *charging*, *discharging* or *frequency regulation* at a particular time. If it does not perform any of these operations at that time, we call this as the *idle* operation state. Time is modeled as a collection of time slots $\mathbb{T}$ (Figure 2). We use $O_{v,t}$ to denote the operation which is performed by vehicle v at a time slot t ($t \in \mathbb{T}$). The parameter O, takes four values: 0, 1, 2, and 3, denoting the operations: idle, charging, discharging, and frequency regulation, respectively.

Price Model. Electricity is sold based on a time-varying price model. We use PE_t to denote this price of a unit of electricity (kWh) at time slot t. In the case of charging, the aggregator follows this price rate to pay the grid, while in the case of discharging, the grid pays to the aggregator in the same rate. The participation in frequency regulation is also paid by the energy provider. We use PF_t to denote the price of a unit of electric capacity (kWh) of the battery that is participating in frequency regulation during time slot t.

Requirements. We know that the energy provider has a number of constraints to be satisfied. NF_t represents the minimum aggregated capacity required for participating in frequency regulation during slot t. XF_t represents the maximum aggregated capacity for which the energy provider will pay if the capacity is used for frequency regulation during slot t. During time slot t, the total amount of discharged energy is required to be limited within the minimum and maximum boundaries which are denoted by the terms ND_t and XD_t, respectively. The parameter XC_t denotes the maximum amount of total electricity that is possible to be consumed by the vehicles during slot t. There are a number of requirements with respect to a subscribed vehicle and the aggregator. A vehicle requires a minimum SOC at the end of the subscription period, EC_v. Each vehicle's battery should have the minimum capacity NB and the minimum charging rate NR.

3.3 Management Modeling Parameters

We use the following parameters in order to model the management plan for the aggregator:

- *Stored electricity at a time slot.* $C_{v,t}$ represents the stored electricity of vehicle v at the end of time slot t. It depends on the stored electric charge at the beginning of that slot (i.e., the stored electricity at the end of the earlier slot) in addition to the operation chosen for the vehicle at that slot (i.e., $O_{v,t}$).
- *Consumed electricity for charging during a time slot.* $VC_{v,t}$ represents the electricity consumed by vehicle v for charging the battery during time slot t.
- *Discharged electricity to the grid during a time slot.* $VD_{v,t}$ is the electricity supplied by vehicle v to the grid by discharging the battery during time slot t.
- *Capacity used for frequency regulation during a time slot* $VF_{v,t}$ is the capacity offered by vehicle v to be used for participating in frequency regulation at time slot t.
- *Total electricity consumed for charging during a time slot.* TC_t is the total electricity consumed for charging the batteries of all subscribed vehicles during time slot t.
- *Total electricity discharged to the grid.* TD_t is the total electricity discharged to the grid during time slot t by all subscribed vehicles.
- *Total capacity used for frequency regulation.* TF_t is the capacity of all subscribed vehicles used for participating in frequency regulation during time slot t.

3.4 V2G Management Model

The V2G management plan for an aggregator is the collection of schedules of operations for all PEVs. In this subsection, we formalize the consumed / discharged electricity and the capacity used in frequency regulation for each vehicle based on the chosen operations. We also formalize the payoff of the aggregator. Finally, we formalize the requirements.

3.4.1 Modeling Vehicle's Operation

We know that a vehicle can select any of the four operations (i.e., 0 to 3 in number) at a particular time slot. The choice of operations for a vehicle is only valid while it is connected to the grid (i.e., during the subscription period). Before and after the subscription period of a vehicle, it is obvious that the vehicle executes the idle operation. These constraints are formalized as follows:

$$(t \geq ST_v) \wedge (t \leq ET_v) \rightarrow (O_{v,t} \geq 0) \wedge (O_{v,t} \leq 3) \quad (1)$$

$$(t < ST_v) \wedge (t > ET_v) \rightarrow (O_{v,t} = 0) \quad (2)$$

If participating in frequency regulation is chosen as the operation for a vehicle during a time slot, the vehicle should have been agreed to participate in this operation. This constraint is formalized below:

$$(O_{v,t} = 3) \rightarrow F_v \quad (3)$$

3.4.2 Modeling Charge at a Time Slot

The stored electricity $C_{v,t}$ of vehicle v at the end of time slot t depends on the electricity/charge stored at the earlier slot, the operation chosen for the vehicle at that slot (i.e., $O_{v,t}$), and the capacity of the battery (B_v). In case of charging operation, the stored electricity increases. However, the stored electricity cannot exceed the capacity. In the case of the discharging operation, the stored electricity decreases until it is zero. In the case of the idle and frequency regulation operations, the stored electricity remains unchanged. It is required to initialize the stored charge at the beginning of the starting time slot (i.e., the stored charge at the previous slot) by the given initial stored electric charge of the vehicle. The following equations formalized the above mentioned constraints:

$$(O_{v,t} = 0) \vee (O_{v,t} = 3) \rightarrow (C_{v,t} = C_{v,t-1}) \quad (4)$$

Let, $CA_{v,t} = C_{v,t-1} + R_v$. Then,

$$\begin{aligned}(O_{v,t} = 1) \rightarrow &((CA_{v,t} \leq B_v) \rightarrow (C_{v,t} = CA_{v,t})) \wedge \\ &((CA_{v,t} > B_v) \rightarrow (C_{v,t} = B_v))\end{aligned} \quad (5)$$

Let, $CS_{v,t} = C_{v,t-1} - R_v$. Then,

$$\begin{aligned}(O_{v,t} = 2) \rightarrow &((CS_{v,t} \geq 0) \rightarrow (C_{v,t} = CS_{v,t})) \wedge \\ &((CS_{v,t} < 0) \rightarrow (C_{v,t} = 0))\end{aligned} \quad (6)$$

$$(t < ST_v) \vee (t > ET_v) \rightarrow (C_{v,t} = C_{v,t-1}) \quad (7)$$

$$(t = ST_v) \rightarrow (C_{v,t-1} = IC_v) \quad (8)$$

The electricity consumed for charging ($VC_{v,t}$), supplied to the grid ($VD_{v,t}$), or used for frequency regulation ($VF_{v,t}$) with respect to a vehicle at a time slot is formalized (Equation 9) according to the operation chosen at that time slot.

$$\begin{aligned}&(O_{v,t} = 1) \rightarrow (VC_{v,t} = C_{v,t} - C_{v,t-1}) \\ &(O_{v,t} = 2) \rightarrow (VD_{v,t} = C_{v,t-1} - C_{v,t}) \\ &(O_{v,t} = 3) \rightarrow (VF_{v,t} = B_v)\end{aligned} \quad (9)$$

All other cases, $VC_{v,t}$, $VD_{v,t}$, and $VF_{v,t}$ are zero. Then, we formalize the summation of the consumed electricity for charging (TC_t), the supplied electricity by discharging (TD_t), and the consumed electricity in support of frequency regulation (TF_t) for all vehicles at each time slot. For example,

$$TC_t = \sum_{v \in \mathbb{V}} VC_{v,t} \quad (10)$$

3.4.3 Modeling Payoff

The computation of the payoff (the cost/price paid/received) P by the aggregator formalized by summing the consumed electricity for charging (TC_t), the supplied electricity by discharging (TD_t), and the used electricity for frequency regulation (TF_t) for all vehicles at each time slot. For the first kind of electricity, aggregator requires to pay to the energy provider, while for the second and third kinds it receives revenue from the provider.

$$P = \sum_{t \in \mathbb{T}} (TD_t \times PE_t + TF_t \times PF_t - TC_t \times PE_t) \quad (11)$$

3.4.4 Modeling of Users' Requirements

There are different user-driven constraints. We consider that the energy provider has constraints on the consumed electricity in charging (TC_t), the discharged electricity (TD_t), and the used electricity for frequency regulation (TF_t). All these constraints are asserted in the following equations:

$$\begin{aligned}&(TC_t \leq XC_t) \wedge (TD_t \leq XD_t) \wedge (TD_t \geq ND_t) \wedge \\ &(TF_t \geq XF_t) \wedge (TF_t \geq NF_t)\end{aligned} \quad (12)$$

$$(B_v \geq NB) \wedge (R_v \geq NR) \quad (13)$$

Each vehicle has a minimum limit on the energy stored in the battery at the end of the participation period.

$$(t = ET_v) \rightarrow (C_{v,t} \geq EC_v) \quad (14)$$

After satisfying all the above mentioned constraints, the main constraint of the aggregator is to receive a minimum benefit (i.e., payoff).

$$P >= NP \quad (15)$$

3.5 SMT Encoding

We encode the system configuration and the constraints into SMT logic [3]. We use the *Z3 .Net API* [14] for encoding the formalization of our proposed management model programmatically.We use mainly *integer* terms for the formalizations. Most of these terms are applied in the forms of *uninterpreted operations*. The uninterpreted operations return integers. The system configurations and the constraints are given in a text file (*input* file). By executing the model (in Z3), we obtain the verification result as either satisfiable (*sat*) or unsatisfiable (*unsat*). If the result is unsat, it means that the problem has no schedule that satisfies the constraints. In the case of sat, we get the management plan from the assignments of the variables, $O_{v,t}$, the operations selected for each vehicle at different time slots. The results from our model is also printed in a text file (*output* file).

3.5.1 Example

We quote a small example in order to delineate our model and its execution. The input corresponding to the example is shown in Table 2. We want to find a model that shows the possible choices of the operations for each vehicle at different time slots that satisfy the given constraints. In order to keep the example small, we consider only 4 vehicles and 8 time slots. The execution of the model corresponding to the example gives a sat result. The important part of the solution (i.e., the assignments to different variables of the model) is shown in Table 3. From the assignments, we find that one of the possible selections of the operations for the vehicle numbered 2 is $\{1, 3, 3, 1, 2, 3\}$ for the time slots 2 to 7, respectively. That is, the vehicle will take charge at time slot 2 and 5. It will participate in frequency regulation at the time slots 3, 4, and 7. It will discharge electricity to the grid at the time slot 6.

Table 2: Input to the Example

```
# Number of vehicles, number of time slots
4 8

# Vehicle information
# (start and end slots, capacity, initial and
# end stored charges, charging rate)
1 4 30 10 20 10
2 7 40 20 30 10
4 6 20 10 20 10
5 8 30 0 20 10

# Electricity price at different time slots
1 1 1 2 2 2 1 1

# Frequency Regulation price at different time slots
1 1 1 2 1 2 1 1

# Max capacity and min capacity, which are required for
# participating in frequency regulation at different slots)
40 40 40 50 40 55 40 40
20 20 20 35 25 35 20 20

# Max electricity and min electricity, which are allowed to
# discharge to the grid at different time slots
30 30 40 30 40 40 30 30
10 10 10 10 20 10 10 10

# Max electricity for consumption by charging at different slots
60 60 50 30 40 40 50 60

# Other constraints
# (min capacity per vehicle, min revenue earned)
10 100
```

3.6 Optimal Solution

The verification result comprehensively represents a consistent V2G management plan for the network satisfying all of the constraints. Usually there are more than one model that satisfy the constraints. These models also give different payoffs, though all of them give payoff more or equal to the payoff constraint (NP). Observing these models, one can choose the best schedule among all satisfiable models (i.e., alternative models) for the same set of constraints. We provide an algorithm (Algorithm 1) that find the optimal V2G

Table 3: Z3 Output (Partial) from the Example

```
.........
VFv -> {
# (<Vehicle ID> <Time Slot> -> <Operation>)
1 1 -> 1
2 2 -> 1
1 2 -> 1
2 3 -> 3
1 3 -> 2
2 4 -> 3
1 4 -> 3
3 4 -> 3
2 5 -> 1
3 5 -> 1
4 5 -> 1
2 6 -> 2
4 6 -> 1
2 7 -> 3
4 7 -> 1
4 8 -> 2
else -> 0 # No (i.e., idle) operation is selected for the rest
}
.........
```

plan based on the payoff constraint.

The algorithm utilizes a binary search method, to find the optimal value. Algorithm 1 usually takes a longer time than the time needed for finding a satisfiable model only, since the algorithm requires several invocations for the model synthesis. The complexity of the algorithm is $O(T_{verify} log_2 D)$, where T_{verify} is the verification time and D is the difference between NP_{max} and NP_{min}. Since T_{verify} is very high in unsatisfiable cases as well as in tight constraint-based cases (see Section 4 for details), for a large number of vehicles the time for finding the optimal would be very high. However, the aggregator is centrally running this optimization, it can utilize powerful machine to compute this optimization. Even, it can control the number of steps (K_{max}) to reduce the optimization time. In this case, the aggregator may receive a close-to-optimal scheduling for V2G management.

4. EVALUATION

In this section, we first present the evaluation results for analyzing the behavior of the management plan with respect to different parameters. Then, we evaluate the proposed model in terms of *scalability*. We analyze the model using different synthetic V2G systems.

4.1 Impact of Different Properties on V2G Management Plan

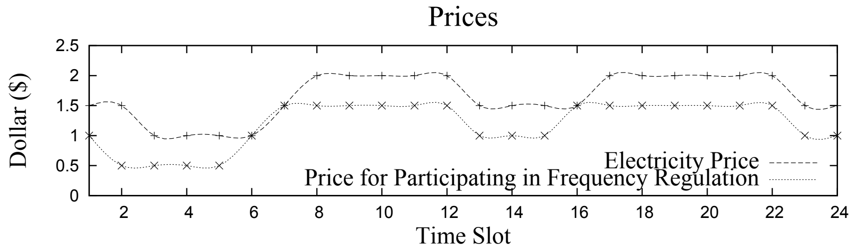
The following two factors have important effects on the V2G management plan: (i) the vehicles' distribution, i.e., the number of vehicles and the subscription period of each vehicle, and (ii) the buying/selling price of electricity and the price (payment) for participating in frequency regulation. Since many other constraints influence the management plan, the effects are not straightforward to observe. In this analysis, we considered a synthetic V2G system of

Algorithm 1 An Algorithm for Finding The Optimal Payoff

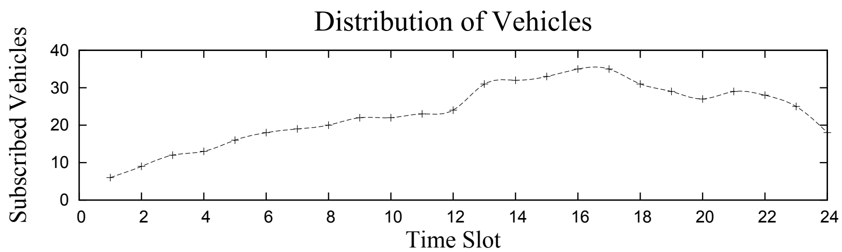
NP is the payoff constraint.
NP_{max} is the maximum possible payoff (L).
NP_{min} is the minimum possible payoff (e.g., $-L$).
K_{max} is the maximum number of executions of the loop-body.
if Solver returns SAT **then**
  Get Model M.
  Update NP_{min} according to the value of P in M.
  $K = 0$.
  repeat
    $NP = (NP_{min} + NP_{max})/2$.
    Update the associated constraint formalization (i.e., Equation 15) based on updated NP.
    if Solver returns SAT **then**
      Get Model M.
      Update NP_{min} according to the value of P in M.
    else
      $NP_{max} = NP$.
    end if
    $K = K + 1$.
  until $(NP_{max} - NP_{min} \approx 0)$ **or** $(K = K_{max})$.
end if

100 vehicles distributed over 24 time slots based on their subscription periods. Fig. 3(b) shows the distribution of the vehicles, i.e., the total number of vehicles subscribed to the system during each time slot. Fig. 3(c) shows the total energy consumption (i.e., due to charging), the total energy sold (i.e., discharged to the grid), and the total capacity used in frequency regulation during 24 time slots. The corresponding electricity price and the price for participating in frequency regulation are shown in Fig. 3(a).

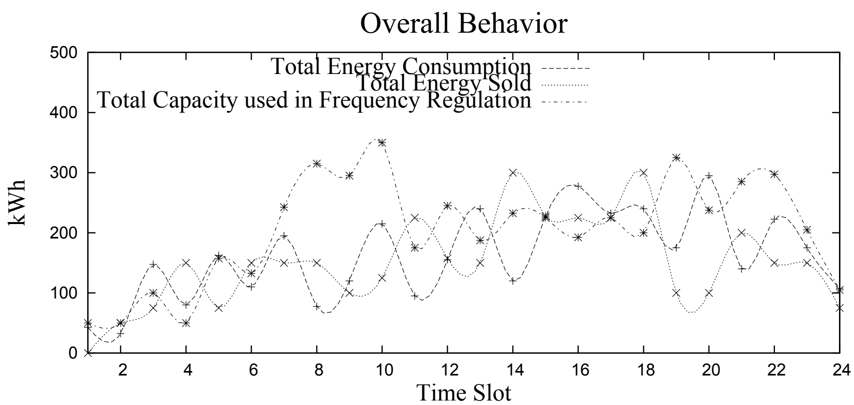
According to Fig. 3(b), the number of subscribed vehicles during the initial time slots (i.e., from time slot 1 to time slot 8) is less comparing to the number of subscribed vehicles during the other time slots. However, the number of subscribed vehicles starts to reduce during the ending time slots. The prices for consuming or selling electricity and participating in frequency regulation follow the same pattern as shown in Fig. 3(a). The prices are high during time slots 7 to 12 and 17 to 22, while they are low during time slots 2 to 6. In Fig. 3(c), we see that the total energy consumption, the total energy sold, and the total capacity used in frequency regulation are low in the beginning of the period, which is due to the less number of subscribed vehicles during that time slots. During time slots 7 to 11, we see that the total capacity used in frequency regulation is significantly higher compared to the total consumed or sold energy. Though the reason of this behavior depends on many constraints, we can explain this as follows. The aggregator can earn money by selling the stored energy or participating in frequency regulation, though the total consumed and sold energy should be within the minimum and maximum bounds. Since the number of subscribed vehicles are limited in this period, the aggregator has little freedom to choose both of selling energy and participating in frequency regulation. Moreover, if stored energy of a vehicle's battery is sold, it has to be recharged up to the required SOC. In addition to that, participating in frequency regulation cannot be very random. The aggregator has to ensure the participation for a minimum duration. As a result, despite the price of electricity is little higher than that of participating in frequency regulation during this period, participating in frequency regulation is chosen more often compared to selling energy. During the



(a)



(b)



(c)

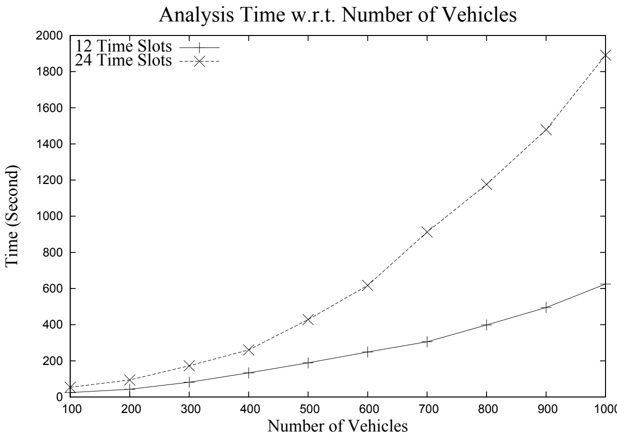
Figure 3: (a) The price of the electricity and that of the participation in frequency regulation with respect to the time slots, (b) the distribution of the vehicles with respect to the number of slots, and (c) the total energy consumption, energy sold, and the capacity used in frequency regulation.

highest availability of the vehicles (at time slots 12 to 16), both of them are chosen almost in the same rate. However, during the ending time slots, selling energy is again reduced, since the number of subscribed vehicles leaving from the system is higher than the incoming vehicles. During the end of the subscription period a vehicle has seldom chance to be selected for selling energy because of its minimum SOC requirement.

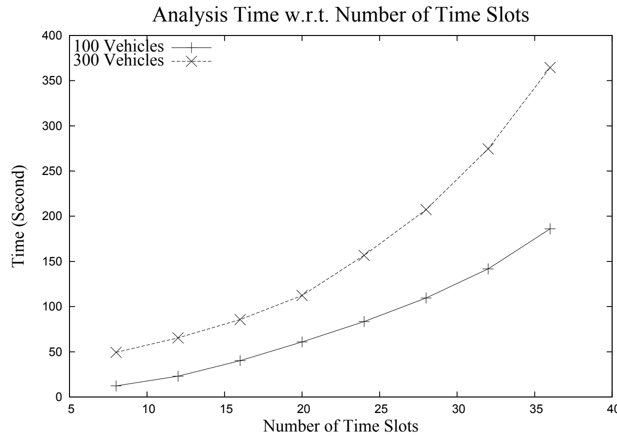
4.2 Scalability

Methodology. We evaluated the scalability of our proposed model by analyzing the *time* and *space* required in constraint verification using different synthetic V2G systems with different problem sizes. Problem size depends mainly on the number of vehicles and the number of time slots. We varied the constraints based on the number of vehicles, especially the constraints on the charging, discharging, and frequency regulation. Since the increase/decrease in the number of vehicles increases/decreases the accumulated electricity charged/discharged/used in frequency regulation at a particular time slot, we increase/decrease the associated constraint values. We varied the number of vehicles within 10 to 1000, while the number of time slots within 8 to 36. We encoded our model using Z3 .NET API and ran the verification of the model on an Intel Core i3 Processor with 4 GB memory.

Impact of the Problem Size. Fig. 4(a) shows the model synthesis time with respect to the problem size. We observed that the analysis time increases (almost quadratically) with the problem size. We varied the problem size with respect to the number of vehicles. We did the experiments taking two different numbers of time slots. We observed that the higher the number of time slots is, the more increase in the execution time. Fig. 4(b) shows this behavior in detail. We also see that the synthesis time increases almost quadratically

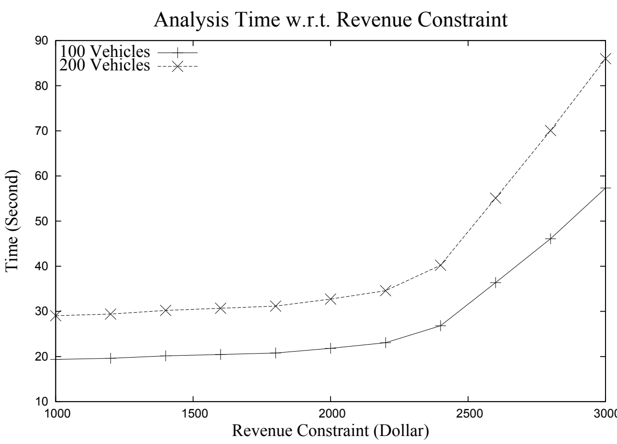


(a)

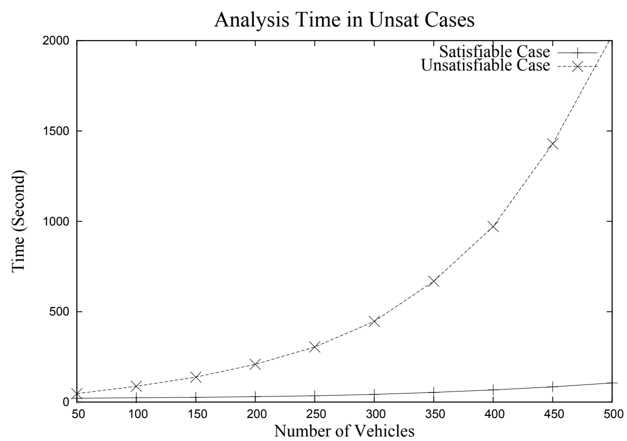


(b)

Figure 4: (a) The model synthesis time with respect to the number of vehicles and (b) the model synthesis time with respect to the number of slots.



(a)



(b)

Figure 5: (a) The impact of revenue constraint on the synthesis time and (b) the model verification time in case of unsatisfiable problems.

with the increase in the time slots.

Impact of the Constraints. We analyzed the impact of the tight or relaxed constraints on the model verification time. In this experiments, we tightened (or relaxed) the profit constraint by increasing (or decreasing) the value. The analysis result is shown in Fig. 5(a) varying the profit constraint value. In this analysis, we considered a fixed number of vehicles (100) and a fixed number of time slots (20). We observed that the execution time increases with the increase in the profit constraint value. This is due to the fact that a larger profit value reduces the number of potential solutions; as a result, the search would take a longer time before a solution is found.

Performance in the Unsatisfied Cases. In the cases of very tight constraints (e.g., very high values for the profit constraints), there may not be any satisfiable solution. In such cases, the SMT solver takes significantly larger time to give the unsatisfiable (*unsat*) results compared to the required time in satisfiable cases. Fig. 5(b) shows such a comparison between the satisfiable and unsatisfiable cases varying the number of vehicles. The reason behind this behavior is that the SMT solver requires verifying all possible ways to conclude that there is no solution based on the given constraints.

Space Complexity. The space (memory) requirement by the SMT solver [14] for our model is evaluated by changing the number of vehicles. The evaluation is done considering the memory (heap size) required for encoding the problem. The required memory for this model verification is the sum of the memory for modeling the system configurations and that for modeling the constraints. The analysis result is shown in Table 4. We observed that the memory requirement increases linearly with the increase in the number of

Table 4: The required memory space with respect to the problem size

Vehicles	Memory (in MB)
100	46.788
200	84.056
300	121.016
400	161.868
500	193.396
1000	386.792
1500	773.584
2000	1105.12

vehicles. If the model size increases significantly, the SMT solver fails to give a solution. An increase in the model size depends on the system size (e.g., the number of vehicles).

5. RELATED WORK

In this work, our main goal is to help the energy provider regulate the production rate to reduce cost and provide stable service. The idea of utilizing the electric cars in providing frequency regulation is a potential but brings many challenges. The researchers looked at those challenges and solved some of them. Their solutions were based on the way they addressed the problem. For instance, looking at the problem from an aggregator perspective is different than when you look at it as a PEV's owner or energy provider. Each party has its own constraints and expectations. Therefore, the researches on vehicle-to-grid (V2G) control algorithms are diversified accordingly. The work in [15] targets the problem of maximizing the profits for the PEV owners by selling excessive energy to the grid. Binary particle swarm optimization is used to determine whether the PEV should be charged, discharged, or in a standby mode. Frequency regulation is integrated with the V2G system in [7]. The PEVs in the V2G system can either be charged or provide frequency regulation. A dynamic programming algorithm is proposed to obtain the optimal control sequence for each PEV. Both of these approaches assume that the future electricity pricing information is given in advance, based on a day-ahead pricing model.

The real-time V2G control problem under price uncertainty is studied in [16]. The authors model the electricity price as a Markov chain with unknown transition probabilities. They formulate the problem as a Markov decision process and apply Q-learning to maximize the profit during the parking time of the PEV. The model works with the assumption that the price is available hourly. The authors in [17] address three constraints: maximal apparent power, charging deadlines, and battery capacity and use convex programming to find the optimal schedule of charging and discharging operations for frequency and voltage regulation.

Different to the works described above, in this paper, we have looked at the V2G problem from the sustainability point of view. We have defined the problem as an SMT logic based constraint satisfaction problem, which considers a comprehensive list of constraints associated with the PEVs, the energy grid, and the aggregator. These constraints are required for managing V2G operations in order to keep the grid sustainable as well as to provide an expected benefit to each participating party. We have also proposed a control mechanism for load balancing, which considers the leverage of charging prices for choosing a navigation plan. Our models are easily extensible for any new option or requirement.

6. CONCLUSION

For the sustainability of the smart grid, safe and efficient management of V2G services is important. In this paper, we have presented a formal model that provides the aggregator with a management plan, i.e., the operations selected for each vehicle at different time slots, that satisfies both of the requirements of the users (the grid and the vehicle owners) as well as the constraints of the aggregator. We have modeled the problem formally in the satisfiability modulo theory. We have encoded the model in Z3, a well-known SMT solver. We ran simulation experiments using different synthetic V2G systems with various sizes and constraints. We evaluated the scalability of our model in terms of time and space requirements. We observed that our model is scalable, both in time and space complexity, for problems with thousands of vehicles. In case of 1000 vehicles and 24 time slots, it took around 1800 seconds. As we explained earlier, our model considers subscribers registering with an aggregator. The energy provider can then deal with one or more aggregators, so, its aggregator responsibility to synchronize subscribers' events, thus,assuming an aggregator has thousands of subscribers is valid. Our evaluation proved the scalability of our model for thousands of subscribers per aggregator. In the future, we plan to address the dynamic and unexpected behavior of the vehicles, e.g., leaving the system before the promised or subscribed period. We would like to extend our solution for the scenario of multiple aggregators working together to achieve an optimal performance. Besides, we will also explore the privacy issues and corresponding solutions on our proposed centralized V2G management.

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