Optimal Deployment of Charging Stations for Electric Vehicles: A Formal Approach

Amarjit Datta*, Brian K Ledbetter*, and M. Ashiqur Rahman
Department of Computer Science, Tennessee Technological University, USA
Emails: adatta42@students.tntech.edu, bkledbette42@students.tntech.edu, and marahman@tntech.edu

***Abstract*—Electric Vehicle (EV) is a great innovation of the modern automobile industry. Due to its attractive cost-efficient features and a growing worldwide environmental awareness, the number of EV purchases is growing at an increasing rate day by day. As the price of EVs is expected to drop in the near future, a large number of new EVs will hit the road consequently. However, our current infrastructure is not capable of supporting this growing number of EVs. We need sufficient number of charging stations, placed optimally across an area, to recharge the EVs in a reasonable amount of time. In this paper, we present a formal framework to optimally deploy charging stations for EVs in a given area. The framework designs this verification as a constraint satisfaction problem where the goal is to optimally place the charging stations, with a sufficient number of charging outlets at each station, to serve all EVs in a given area while satisfying the limited budget and other system constraints. We evaluate the proposed framework for its analysis capability as well as its scalability by performing simulation on different synthetic test cases.**

***Index Terms*—Electric vehicle; charging station; optimal deployment; formal modeling**

I. Introduction

Electric Vehicles (EVs) are the future of the automobile industry. EV is promoted as a viable vehicle technology that reduces the dependence on conventional fossil fuels, thus limiting the greenhouse gas emission [1]. Since the EVs are significantly cost-effective compared to the traditional gasoline vehicles and there is a growing worldwide environmental awareness on the greenhouse effect, the trend of EV purchases is on the rise. U.S. runs the largest number of EVs in the world, with over 506,450 highway-capable lightweight EVs [2], [3]. As of May 2016, the U.S. EV stock represents 30.7% of the global stock of light-duty EVs. California is the largest regional EV market in U.S. with over 223,000 registered plug-in EVs (as of June 2016) [4]. As the government is taking different initiatives to promote EVs to the mass market by creating social awareness against greenhouse emission and offering incentives like tax reduction, free registration, etc., the number of EVs on the roads is growing rapidly.

Although the growing number of EVs is promising, our current charging infrastructure is not capable enough to handle such a high number of EV requests. With more electric vehicles on the road, the existing infrastructure will not be able to support the increase. California has already started to face this problem [5]. Three main problems seem to emerge with the current infrastructure. Firstly, there are simply not enough charging stations in some areas. Secondly, the charging stations that we currently have are small in number to support the influx of new EVs, thus resulting in longer wait times for the EV owners to recharge. Lastly, some areas do not even have a public charging station, thus leaving EV owners with no suitable way to recharge their vehicles. We need an optimal layout plan for deploying necessary charging stations and outlets for serving existing and upcoming EVs.

There are often many locations in an area where it is feasible to deploy charging stations. However, different locations often have different (leasing) cost, while some locations can already be preoccupied. There are usually different types of EVs with different battery capacities. The total cost required for deploying the charging station infrastructure must be within the available budget. The overall deployment cost is the sum of three different costs: (1) station construction cost, (2) infrastructure cost, and (3) maintenance cost. There are also time constraints. An EV owner cannot wait for hours till his or her turn to be recharged. The problem of placing the charging stations and identifying its properties (i.e., outlets) on different locations based on the number of EVs, their properties, time constraints, and limited budget is a mathematically complex, multi-objective problem. We want to determine an optimal solution to this problem that require minimum budget and can serve maximum number of EVs. In this paper, we propose a formal framework to provide an optimal solution to this problem. Our formal framework models the charging station properties, EV properties, location properties, budget constraints, and quality of service requirements using mathematically and provide an optimal layout plan for the stakeholders to deploy the charging stations (and corresponding outlets). The proposed formal framework is modeled as a constraint satisfaction problem, which is implemented using an efficient Satisfiability Modulo Theories (SMT) solver [6]. We provide multiple case studies to illustrate the execution of the proposed framework. We also evaluate the framework in terms of its analysis capability and scalability.

The rest of this paper is organized as follows: In Section II, we provide our motivation and contributions. We propose the synthesis framework in Section III. The formalization of the framework is briefly discussed in Section IV. In the following section, we describe an example case study. Evaluation results are discussed in Section VI. We briefly discuss the related work in Section VII and conclude the paper in Section VIII.

*Both authors contributed equally to this work.

II. Research Motivation and Contributions

A. Research Motivation

EVs are becoming more and more popular nowadays. As the prices of fossil fuels are increasing and people are becoming more aware of the impact of global warming, they have drawn attention towards the use of EVs as an alternative mode of commute. Giant vehicle manufacturer companies like Tesla, Toyota, Honda are investing a great fortune in EV development [7] [8] [9]. Since EVs are being manufactured by these different companies, they have different battery capacities, the maximum distance they can travel, and charging speed. As the number of EVs is increasing, we need to build more charging stations to provide services in a given region.

In a area, an EV can travel from a one source point to another destination point using any of the possible travel routes. When an EV is traveling, there can be different situations. If the EV has enough battery charge, it can reach its destination directly without stopping for recharge. An EV can choose to recharge at any charging station, even though it has enough charge to reach the destination. If the battery charge is not sufficient, it must recharge itself from one (or more than one) intermediate station. Finally, in a chosen station, an EV may need to wait, if the station is occupied (all outlets are busy). Stations have limited number of outlets.

Charging station placement problem is a complex decision problem. A solution of the problem will place the charging stations in appropriate intermediate points to cover the EVs in the area. Since EVs can have different battery capacities, starting locations, destinations, and remaining charge amounts, their requirements will be different. The charging stations can be of different sizes (depending on the number of outlets). A charging station will have different costs based on the size of the station, the number of outlets, and the cost of the location (e.g. lease or rent). Furthermore, the size of the charging station will limit the number of charging outlets, thus fixing the number of vehicles that can charge at a given time. On the other hand, EVs may either decide to charge completely or partially based on the time requirements to reach its destination. In the target area, we want to identify an optimal layout of charging stations, that can serve required number of incoming EVs. A solution to this problem can answer following questions:

- In which points we can place charging stations? Not all points are available. Some points can be infeasible/preoccupied.
- How many outlets each charging station will have? Not all locations are equally busy. Based on the volume of incoming EVs, some charging stations will require more outlets than others. We do not want to install more outlets than needed as each outlet incurs a cost.
- How many EVs we can support using this layout plan? We want to support the maximum number of EVs. However, EV route patterns are not static (it can change frequently, even in every hour). Also, the total number of EVs traveling in an area can change during the time of the day. We want to find an optimal solution, that considers, the dynamic changing behavior of the EV routing pattern.
- Can we successfully serve a target percentage of EVs with this layout? We may not be able to serve all routes. As a solution to this problem, we want to identify a generalize solution, that serves at least the target percentage of EVs within the limited budget.

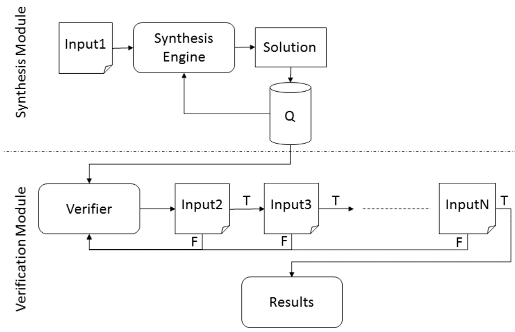


Fig. 1. Synthesis Framework

Placing the charging stations optimally to cover the EV charging requirement and satisfying the budget constraints is a complex problem. Considering all above conditions, optimal placement of charging stations becomes a complex NP-hard problem where the goal of the solution is to find the optimal number of charging stations with different size capabilities in a given area while satisfying all EVs' charging requests, time constraints, and travels to their respective destinations with the overall system cost within the budget constraint.

B. Contributions

In this paper, we try to solve the optimal charging station placement problem using an efficient formal framework. The paper is motivated from this stance leading to the following two key contributions:

- Formal Framework: We propose a formal framework to find an optimal solution of this Np-hard problem. This framework considers EV properties (battery capacity, current battery charge), charging station properties (maximum number of outlets a station can hold), budget limitations, and EV travel route properties. To make realistic assumptions of the EV travel routes, we also consider the number of EVs and their routing path as a function of time during a day. We implemented our framework using SMT [6], powerful problem solver tool that takes problem definitions/constraints as input in formal language (first-order logic) and provides solutions within reasonable time. We model the problem parameters logically in SMT and solve the model using Microsoft Z3 (an efficient SMT solver) [10].
- Comprehensive Evaluation: We present a comprehensive evaluation of our proposed framework by analyzing its solution penetrating and scalability capability.

III. Synthesis Framework

In our proposed synthesis framework, we take EV properties, charging station properties, location properties, costs, and budget limitations as input and generate an optimal placement plan for the stations, which can guide the stakeholders. In a given area, the number EVs traveling and the routes they follow can change frequently. During different times of the day, there may be different number of EVs on the road. Usually, 6 - 9am is the busy hours (people commute to their offices and schools). From 9am to 5pm, there are usually less traffic. From 5pm to 8pm we may again see busy traffic on the road. After 10pm, roads are more vacant. In order formulate this behavior properly, we must consider different traffic behaviors of the day (6am - 10pm). In our framework, we split the entire traffic (6am - 10pm) into multiple time slots and find a generic solution that satisfies all slots. We can see the architecture of the synthesis framework in Fig 1. Our framework comprises of two main modules:

- Synthesis Module: This is the core module of our framework. In the module, we provide EV properties, location properties, and budget constraints as input. Synthesis module also take the first time slot (first traffic pattern, from 6am - 8am) as input. It then generates solutions for that time slot and store that in a queue (Q). In this framework, synthesis module is responsible for traversing all the possible solutions and share that to the next module for further verification.
- Verification Module: This module runs in parallel with the synthesis module. In this module, we also take same EV properties, location properties, and budget constraints as input (same as previous module). Moreover, verification module also take the traffic patterns of other time slots (remaining $N-1$ time slots, if we have total N slots) as input. Iteratively, it takes one solution from the queue and verifies it against all other remaining traffic patterns (of other time slots). If it finds a solution that satisfies every time slot traffic behavior, it is stored as a candidate generic solution. Verification module then compare the generic solutions and determine the optimal solution (that requires minimum budget and serves maximum EVs).

IV. Formal Model of the Charging Station Placement Framework

In this section, we present the formal model of our framework. In order to present our model, we need a number of parameters to denote the EV properties, charging station properties, and other constraints. We present some of the important parameters in Table I. In this paper, no multiplication of two parameters is performed without the multiplication sign ($\times$).

TABLE I
Important Modeling Parameters

Notation	Definition
$start_i$	Starting point of EV_i.
$dest_i$	Destination point of EV_i.
$budget$	Total budget.
EV_i	ith EV.
ST_j	Charging station at point j.
D_{p_x,p_y}	Road distance between point p_x and point p_y.
D_i^{total}	Total distance traveled by EV i.
$CRG_{i,j}^{choice}$	Charge choice for the EV_i at station in point j.
$TIME_i^{max}$	Maximum time the EV_i is allowed to be in the system.
$TIME_i^{act}$	Actual time needed to charge by the EV_i.
$m_j^{station}$	Whether point j has a charging station.
m_j^{chosen}	Whether station in point j is chosen.
$const^{m,crg}$	Miles to charge factor.
$const^{d,t}$	Distance to time factor.

A. Preliminaries

In a typical area grid G, there can be P points and N^{ev} electric vehicles. Each EV_i is trying to reach its destination point $dest_i$ from its starting point $start_i$. The initial charge of the EV_i is I_i. To travel from the starting point to the destination point, the EV_i could take a variety of different paths. Let $\mathcal{R}_{x,y}$ be the set of all possible paths between point x and point y. In this framework, we assume that an EV can only travels to a single route in a time slot. Here, we divide the entire day into multiple time slots where an EV may travel in a different route on a different time slot. In our framework, we are not concern of any specific EV, rather we want to focus on a generic solution that can serve a large number EVs.

B. Station Properties and Budget Constraints

A station ST_j can be of type S_j^{type}. Each type can have a maximum MAX_{type}^{outlet} outlets. Depending on the size of the charging station, a station ST_j can contain N_j^{outlet} number of outlets. The number of outlets in a charging station ST_j must be at least equal to one and less than or equal to the maximum number of allowed outlets (MAX_{type}^{outlet}). We can represent this condition using Equation 1.

$$1 \leq N_j^{outlet} \leq MAX_{type}^{outlet} \tag{1}$$

In order to determine the total amount of electricity a charging station can sell ($S_j^{capacity}$), we can multiply the number of charging outlets N_j^{outlet} with the amount of electricity an outlet can sell during a time slot (E^{outlet}). We can formalize this using following equation:

$$S_j^{capacity} = N_j^{outlet} \times E^{outlet} \tag{2}$$

Parameter $C_j^{d,st}$ is the deployment cost of the station (leasing cost for the land). In the station ST_j, there are N_j^{outlet} number of outlets, each have a deployment cost C_j^{deploy}. Then the total deployment cost for the entire charging station (considering both the leasing cost and the outlet installation cost) can be represented using following equation:

$$STCost_j^{deploy} = C_j^{d,st} + (N_j^{outlet} \times C_j^{d,outlet}) \tag{3}$$

Here, $STCost_j^{deploy}$ is the total deployment cost of the charging station ST_j.

In addition to deployment cost, each station will also have a total maintenance cost $STCost_j^{maintenance}$. This total maintenance cost can be obtained by multiplying the total number of chosen outlets in the station (N_j^{outlet}) with the maintenance cost per outlet ($C_j^{m,outlet}$) and summing that with the maintenance cost of the station property ($C_j^{m,st}$) (for example: land maintenance cost). This can be formalized as follows:

$$STCost_j^{maintenance} = C_j^{m,st} + (N_j^{outlet} \times C_j^{m,outlet}) \tag{4}$$

The total cost of the station ST_j is the summation of the deployment cost (Equation 3) and the maintenance cost

(Equation 4). This can be formalize using following equation:

$$STCost_j^{total} = STCost_j^{deploy} + STCost_j^{maintenance} \quad (5)$$

Using Equation 5, we can calculate the total cost for all stations in the given area. This total cost C^{total} must be less than the budget. We can represent this budget constraint using following equations:

$$C^{total} = \sum_{j \in \mathcal{S}} STCost_j^{total} \quad (6)$$

$$C^{total} \leq budget \quad (7)$$

In our framework, our goal is to identify a generic solution, for which C^{total} is minimum.

C. EV Constraints

Each EV_i has a starting point $start_i$ and a destination point $dest_i$. There can be multiple paths between $start_i$ and $dest_i$. In this framework, we do not emphasize on path selection mechanism. Rather, we assume that, an EV is traveling through a path from a source point to a destination point and this path is fixed in a given time slot. In future, we have plans to expand our framework and include multiple paths and path selection algorithms.

Each EV_i has an initial charge I_i. Parameter $B_i^{capacity}$ (kW) is the battery capacity of EV_i. Therefore, we can represent attributes of EV_i using following equation.

$$Attr_i = \{start_i, dest_i, t, I_i, B_i^{capacity}\}. \quad (8)$$

In its path, an EV_i travels from one point to another point until it reaches its destination. When an EV_i reaches a point p_j, our framework calculates how much charge EV_i had before the point and how much change it has after reaching the point. Parameter CRG_j^{before} denotes how much charge EV_i has before coming to p_j and CRG_j^{after} denotes how much charge it has after leaving p_j.

If an EV_i is traveling from point p_x to point p_y, parameter $path_{x,y}$ denotes the chosen path between the source and the destination point. In order to calculate the remaining charge of EV_i (after traveling through $path_{x,y}$), we can use following equation:

$$CRG_y^{before} = CRG_x^{before} - (D_{p_x,p_y} \times const^{m,crg}) \quad (9)$$

Here CRG_x^{before} is the remaining charge of EV_i at point x, D_{p_x,p_y} is the distance between point p_x and point p_y, $const^{m,crg}$ is the factor (constant) that converts miles traveled to electricity consumed, and CRG_y^{before} is the new remaining charge of EV_i after reaching point y.

Within the EV routes, there can be more than one charging stations. In our problem area grid, not all are available for placing charging stations. Some points can be preoccupied. Parameter $occupied_j$ is a boolean constraint that denotes whether the point p_j is occupied. We can only place a charging station at p_j if the point is not occupied. We can represent this using following equation:

$$m_j^{station} \rightarrow \neg occupied_j \quad (10)$$

When EV_i is at point p_j, it can choose to recharge only if there is a active station at that point. If parameter m_j^{chosen} denotes whether the station at point p_j is chosen or not by EV_i for charging, we can formulate the condition using the equation below:

$$m_j^{chosen} \rightarrow m_j^{station} \quad (11)$$

When traveling, an EV_i may find an active charging station at point p_j. It will take charge from the charging station if the remaining charge of EV_i is to sufficient to reach the next charging station (at point p_{j+1}). If parameter $CRG_{i,j+1}^{current}$ denotes the charge of EV_i if it reach the point p_{j+1} without recharging at p_j, then we can formalize the condition whether EV_i will choose point p_j for charging or not as follow:

$$m_j^{chosen} \rightarrow m_j^{station} \wedge (CRG_{i,j+1}^{current} \leq 0) \quad (12)$$

Here in Equation 12, the condition states that an EV_i will choose the charging station at point p_j only if there is an active station at point p_j and the current remaining charge of the EV_i is not sufficient to go to the next charging station at point p_{j+1}.

When EV_i choose to recharge at point p_j, it can choose any of the available charging choices (EV can decide to recharge full or half or any other desired amount). If parameter $CRG_{i,j}^{choice}$ denotes the charging choice of EV_i (numerical value), the we can calculate the new remaining charge of EV_i (after taking charge at point p_j) using following equation:

$$m_j^{chosen} \rightarrow (CRG_j^{after} = CRG_j^{before} + CRG_{i,j}^{choice}) \quad (13)$$

Furthermore, an EV must never run out of charge. We can formalize this as follows.

$$CRG_j^{before} > 0 \quad (14)$$

$$CRG_j^{after} > 0 \quad (15)$$

D. Time Constraints

An EV_i must get through the entire system in a reasonable amount of time. In our framework, the actual travel time is fixed (we do not consider any traffic congestion). The time an EV can spend in the charging station is variable (based on how many vehicles we have in the charging station). We want to minimize the time so that the total required time (for traveling and charging) is below the maximum allowed time.

To calculate the maximum time an EV_i is allotted in the system, we take its total distance traveled through the system (from $start_i$ to end_i) and multiply it with the distance-to-time factor $const^{d,t}$.

$$TIME_i^{max} = D_i^{total} \times const^{d,t} \quad (16)$$

Parameter $const^{d,t}$ represents the expected time an EV owner may spend recharging along with the stoppage overhead (for pulling into the station, paying for services, etc.) in hours per mile. For example, if an EV's travels total 100 miles, its expected charging time would be calculated by multiplying the 100 miles by the factor resulting in the amount of time, in hours, the EV is expected to spend with charging related activities. Note that this factor does not consider travel time through the grid. Let ECT be the estimated charging time in hours. Then the value of distance-to-time factor $const^{d,t}$ can be calculated as follows:

$$const^{d,t} = \frac{ECThours}{mile} \quad (17)$$

To calculate how much time an EV actually takes to charge in a charging station, we need to consider the following three main parameters:

- $T^{overhead}_{i,x}$: The amount of time an EV_i must spend during its xth stoppage for simply stopping, pulling into the station, and paying.
- $outlet^{speed}_j$: The outlet charging speed per kWh at the point p_j.
- $bought_{i,j}$: The amount of kWh purchased by an EV from a charging station at point p_j.

The actual time an EV_i takes in the entire system can be calculated using following equation:

$$TIME^{act}_i = \sum_{x \in \mathcal{X}_\rangle} T^{overhead}_{i,x} + (bought_{i,j}/outlet^{speed}_j) \quad (18)$$

This required time is the time an EV_i spends in the system. This time must be less than the maximum allowed time an EV can spend in the system. We can formalize this time constraint using following equation:

$$TIME^{max}_i \geq TIME^{act}_i \quad (19)$$

E. Adaptable Deployment for Future Scenarios

As the problem parameters (volume of the EV traffic, travel routes, costs, or budget) can change anytime, our framework should be able to adapt itself to the changing conditions. If this framework, we can formulated the conditions in such a way that, we can easily adapt the changing behavior of the parameters.

As an example, if the budget of the problem increases, there will be more opportunities for the framework to install more stations and outlets. Let's assume that, from our previous solution, we already have some charging stations placed in the area. In the budget increases, we can accommodate the change by considering the fact that we already have some stations (in the formal model, we can make those station points active) and only determine new positions, where we can add new stations by using the extra budget. Equation 20 is used for budget constraint. In the new budget is $\overline{budget}$ and the budget difference is $\Delta budget$, then we can change Equation 20 in the following way to accommodate the budget change:

$$C^{total} \leq \Delta budget \quad (20)$$

V. Case Study

In this section, we present two example case studies. In both of the studies, we consider same problem area (an area with 25 location points). In these studies, we use synthetic data. Based on the real world traffic volume, we generate synthetic test data for different times of the day. As discussed in II-A, we divide the entire day traffic data (from 6am to 10pm) into multiple time slots. In these case studies, we consider total 8 slots (2 hours each).

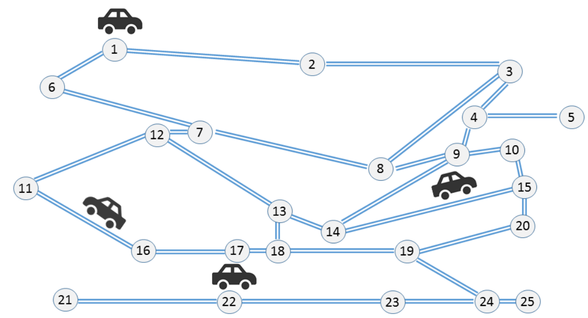


Fig. 2. An example problem with 25 points

TABLE II
INPUT OF EXAMPLE CASE STUDY

Number of EVs, Number of Points, Type of Stations, and Type of Locations 800 25 3 3
Station Type, Maximum Outlets, and Infrastructure Cost($) 1 3 20000 2 5 30000 3 8 40000
Location Number, Preoccupied, and Lease Amount ($) 1 0 500 2 0 500 . . . .
Initial Cost of Setting Up Outlet ($), Outlet Maintenance Cost ($), and Outlet Charging Speed (kW/hr) 1000 50 24
Number of roads 30
Road Number, PointA, PointB, and Distance (miles) 1 1 2 10 2 2 3 10 . . . .
EV ID, Starting Location, Destination Location, Battery Capacity (kW), and Initial Charge (%) 1 1 21 20 20 2 2 15 18 30 . . . .
Distance (miles) to Charge Ratio and Distance (miles) to Time Ratio 0.25 0.015
EV Stopping Cost ($) 0.02
Time Slot Length in Hours and Budget ($) 3 200000

A. Case Study 1

In this framework, the synthesis module determine all possible solutions to the traffic requirement of time slot 1 and the verification module verifies each solution with the traffic requirements of other time slots (time slot 2 to 8). Both verification module and the synthesis module take EV properties, station properties, budget constraints, and time constraints as input. The complete input of time slot 1 is shown in Table II. The example problem area can be observed in Figure 2. In this case study, in each time slot, we consider a different number of EVs, which are traveling in different routes. Station properties, time, and budget constraints are same in all time slots.

In the case study, the time slot 1 to 8 have 800, 700, 600, 500, 400, 600, 800, and 400 EVs respectively. In each time slot, the travel routes of EVs are different (we do not focus on any particular EV). The number of EVs and their travel routes are the only differences between the time slots. All other properties in all the time slots are some. We consider 3 types of station. Each type of station can accommodate different number of charging outlets and incur different infrastructure cost. There are 12 possible locations (among the total 25 points) where we can place stations. Remaining 13 points are occupied (our framework cannot consider these points for station). Every position has a lease amount ($). A station must have one or more charging outlets. The installation charge and the maintenance cost of one outlet is $1,000 and $50 respectively. Outlets have 24 kWh charging speed.

We have 30 interconnected road segments, each connecting two location points. For example, the road segment 1 is connected with location point 1 and location point 2 and the length (distance) of the two points (on this road) is 10 miles. In this case study, the budget limitation is $200,000. The target of our framework is to identify an optimal solution, that can satisfy the traffic behavior of all time slots and provide the placement of the stations, station types, and the number of outlets in the stations. The optimal solution is optimal based on two key attributes: (1) minimum budget needed and (2) maximum number of EVs served.

In order to determine the amount of charge needed to travel a certain distance, our framework considers multiple constraints. In the example, the distance to charge ratio is 0.25. If EV 1 wants to travel from point 1 to point 6 (3 miles distance), it will require to take 0.75 kW charge (3×0.25). Similarly, the distance-to-time ratio value for this example is 0.015. With all above constraints, the execution of the model returns a SAT (Satisfied) result, along with the following assignments of the different variables:

- Our framework can find a satisfiable solution by placing the charging stations at points 6, 9, 12, 13, 17, and 23. The cost of establishing station on these points are $22,050, $22,050, $22,050, $22,050, $47,800, and $47,800, respectively. The number of outlets in these stations are 1, 1, 1, 1, 6, and 6, respectively.
- In this example, the budget constraint is $200,000. The solution found by the framework requires $183,800. The total execution time of our program is 6012.67 seconds, which is roughly 1.67 hours.

B. Case Study 2

In the second case study, we use the same synthetic input data that we used in the case study 1. The only difference between case study 1 and 2 is the budget. In our problem model, the budget is an important constraint. Our framework tries to identify an optimal generic solution that costs within the given budget. If we increase the budget, our framework will have more liberty to place charging stations (with a higher number of outlets) in more locations. Moreover, if we decrease the budget, there will be less option available to the framework to position the charging stations and its outlets. If we reduce the budget too far, we may not find any satisfiable solution at all. In this study, we reduce the budget to $100,000. This budget is much less than the budget in study 1. With this harder budget constraint, the execution of the model produces a $UNSAT$ (Unsatisfied) result. Our framework could not identify a solution that serves in the incoming EVs of different time slots within the tight budget and other constraints.

VI. Evaluation

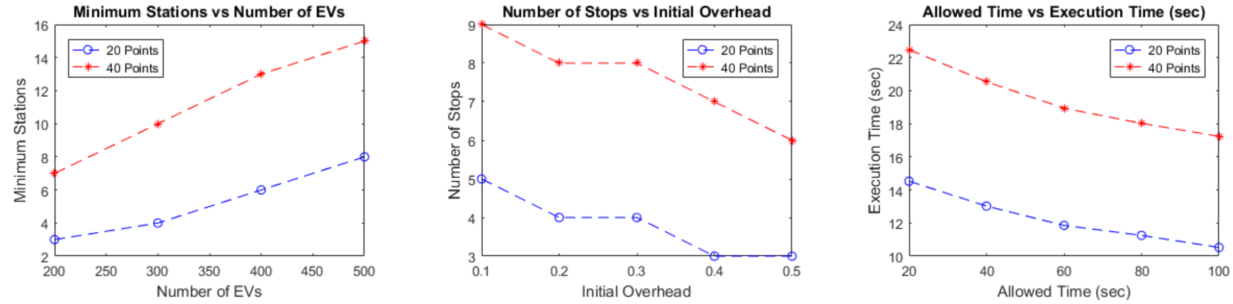
A. Methodology

From our experiments, we analyze the satisfiable placement of EV charging stations in a given problem area, with respect to the total available budget, the number of EVs (during various time slots), and other properties / constraints. We performed this analysis over two different sized areas; one had 20 location points and the other had 40. In the 20 point location area, 11 points are infeasible (we cannot place any charging station of those points). Similarly in the 40 point location area, 15 points are infeasible. In the scalability analysis, we analyze the execution time of our framework with respect to different number of EVs, size of the area, and budgets. For the scalability analysis, we analyze the performance of our framework against larger problem sizes (large number of EVs and bigger area). We run our experiments on an Intel Core i7 Processor PC with 16 GB memory.

B. Framework Analysis

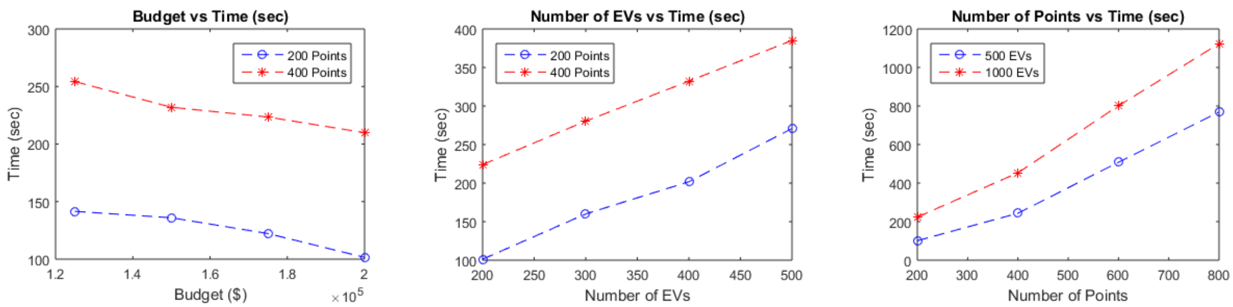
1) Impact of Number of EVs on Synthesis: As shown in Fig 3(a), when the number of EVs increases, the minimum number of stations required also increases. From Fig 3(a), we can observe that, as the number of EVs increases, the minimum number of stations required to solve the problem also increases in linear order. For problem size with 40 points, we need more charging stations in the intermediate nodes than that of the 20 points to make sure that the EVs can reach their destinations without running out of battery charge. As the number of EVs increase, congestion in the station for taking charge also increases. Hence, our framework need to deploy more stations to satisfy the time constraints (maximum allowed time an EV can spend charging in our framework).

2) Impact of Initial Overhead on Synthesis: From Fig 3(b), we can observe the relationship between the initial overhead of the problem model and the number of stops taken by the EV for recharging. As the initial overhead increases, our framework identifies solutions that require less number stops (due to the strict time constraint). When the initial overhead increases, the number of stops taken by the EVs also decreases. The initial overhead is the time spent by the EV to perform miscellaneous tasks during its stop in the charging stations. Here, miscellaneous tasks include the time it takes to park the vehicle at station, plug it in, pay, and leave (everything except the time needed for charging). From the Fig 3(b), we can observe that for 40 points area size, the number of stops required is higher than that of the 20 points area size.



(a) (b) (c)

Fig. 3. These graphs show the analysis of our framework based on the number of EVs, initial overhead, and allowed time: (a) the minimum number of stations required with respect to the number of EVs, (b) the number of stops taken by the EVs with respect to the initial overhead, and (c) the required execution time with respect to the allowed charging time per mile.



(a) (b) (c)

Fig. 4. These graphs show the scalability analysis: (a) execution time with respect to the budget of the problem, (b) execution time with respect to the number of EVs, and (c) execution time with respect to the problem size (number of points in the grid).

3) Impact of Allowed Time on Synthesis: From Fig. 3(c), we observe the relationship between the total allowed time for charging per mile and the execution time. As the allowed time for charging increases, the amount of time an EV can spend at charging stations also increases. As a result, EVs can wait longer in the station (when it is full) for its turn rather than moving to another station. With this liberty, our framework can identify a satisfiable solution within lesser time. In Fig 3(c), we observe that as the allowed time to charge increases, the execution time of our program decreases. From the figure, we can also observe that, the execution time of 40 points system is much higher than the execution time of 20 points system which is obvious since the former model is significantly larger than the later problem model.

C. Scalability Analysis

1) Impact of Budget on the Execution Time: From Fig 4(a), we can observe the execution time of our framework with respect to the budget limitation. In this figure, budget is the combination of 3 costs (deployment cost, infrastructure cost, and maintenance cost). As the budget increases, our framework gets more flexibility to deploy more charging stations and charging outlets. As a result, our framework can identify an optimal solution in less time. From Fig 4(a), we can observe that as the budget increases, execution time of the program decreases. In this figure, we compare the execution time of areas with 200 and 400 points size. Execution time of 400 points grid is higher than the 200 points grid, as the former problem model is significantly larger than the later one.

2) Impact of Number of EVs on the Execution Time: We observe the relationship between the number of EVs and the execution time of our framework in Fig 4(b). For this experiment, As the number of EVs increases, the amount of computation needed by our framework to satisfy all EV charging requirements also increases. If we keep the problem size same and only increase the number of EVs, it increases the computation complexity of our framework. With more EV, we need more charging station (or outlets) to serve the larger traffic. However, as other constraints remains same (same budget, same location pattern), finding an optimal solution requires more computation. As a result, as the number of EVs increases, our framework requires more time to identify an optimal solution. In this experiment, we compare the behavior on 2 area sizes (200 points and 400 points). From the figure, it is visible that the execution time of the 400 points problem area is higher than the execution time of the 200 points problem area (for the same number of EVs).

3) Impact of Area Size on the Execution Time: From Fig 4(c), we can see the relationship between the area size of the grid (the number of points) and the execution time of the synthesis framework. From the figure, we can observe that as the area size increases, the execution time of the framework also increases. When the framework executes on a larger problem area, it requires more time to analyze the area. In this experiment, we compare the behavior with two different number of EVs (500 EVs and 1000 EVs). For the same problem area size, the problem with 1000 EVs take longer execution time than the problem with 500 EVs.

VII. Related Work

Charging station placement problem is a synthesis genre of mathematical problem. In the literature, we can identify multiple approaches that provide solution to this complex problem. In [11], Sebastiani et al. provided a solution to this problem with a simulation optimization technique that used a model of the urban traffic, where they proposed an optimal solution, concentrating on the charging station waiting time. In [12], Mehar et al. proposed a heuristic based solution to the problem using a genetic algorithm. In their paper, the authors presented an optimized algorithm named the OLoCs (Optimized Location Scheme for electric charging stations), which can find an optimal placement of the charging stations in a given area and plots their position on a graph. In [13], Ge et al. proposed a method of locating and sizing of charging station for electric vehicle based on grid partition. Also in [14], the authors applied hierarchical clustering for allocating Battery Electric Vehicle (BEV) stations in the urban areas. Similar to the charging station placement problem, Aslam et al. in [15] discussed the optimal placement of the Roadside Units (RSU) as an important synthesis problem in the Vehicular Ad Hoc Network (VANET).

EVs have limited battery capacity. Minimizing the overall trip time of an EV in the road can be very useful. Baum et al. in [16], proposed a shortest path selection solution for EVs considering realistic models of charging stops, charging powers, battery capacities, and battery swapping stations. In [17], Gerding et al. proposed an online reservation approach focusing on EV charging where an EV owner can request a reservation to a charging station on a certain time slot and location, for the desired amount of charging and once the charging station reports its availability and cost, the EV owner can decide to reserve his slot and make the future plan for recharge. In [18], Lee et al. proposed a DC charger selection scheme for EVs using a variant of traveling salesman problem.

Although the above approaches are interesting, none of them focused on providing an optimal solution to this NP-hard synthesis problem in the stakeholders perspective. In our proposed approach, we try to provide an optimal solution that considers realistic traffic behaviors of the problem area during different time slots of the day while satisfying the budget, system, and time constraints. In our framework, we consider many practical conditions like EV properties, EV routes, traffic volume (during different times of the day), maximum service time constraints (quality of service), station types, location costs, and various infrastructure and miscellaneous costs. In this work, we propose a formal framework to synthesize the placement of the charging stations and considered all above realistic constraints, which is unique of its kind to the best of our knowledge.

VIII. Conclusion

The number of EVs on the road are increasing rapidly. To support this increasing number of EVs, we need an efficient deployment plan of the charging stations that can serve the vehicles within a layout budget. We propose a formal framework to synthesize the placements of the charging stations by modeling the system constraints, budget limitations, and EV charging factors. We take the traffic patterns and a distribution of various EV properties (e.g., battery capacity, initial charge) to model the deployment of the charging stations such that each EV can reach from source to destination within a expected time frame. We conduct simulation to analyze the performance of the deployment synthesis model. We also evaluate the scalability of the formal model. The analysis results proves the efficiency of the proposed solution in solving the deployment problem. In the future, we would like to expand our framework by considering the traffic congestion and a larger set of EV properties other than the charging capacity, along with providing a charging-price control mechanism through a real-time traffic monitoring system.

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